Efficient Calculation of Matrix Exponential Function and Vector Product

Zhu Chaowu

Sanmenxia Vocational and Technical College, Basic Department, Sanmenxia, Henan, China

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Abstract: in Recent Years, Matrix Exponential Functions Have Been Widely Used in Semi-Linear Ordinary Differential Equations and Are Widely Used. Moreover, the Application in Actual Production and Life Has Gradually Increased. among Them, Engineering Mechanics, Molecular Dynamics, Mechanical Design, Etc. through the Matrix Function with Exponential Properties Can Promote Its Application in Actual Production and Provide a Powerful Driving Force for Social and Economic Development. Based on This, This Paper Constructs a Block Krylov Subspace, and Studies the Validity of Exponential Property Matrix Functions and Vector Products, in Order to Provide a Theoretical Reference for the Calculation of Exponential Class Matrix Functions in Daily Life.

1. Introduction

1.1 Literature Review

In the thinking about the calculation of the matrix exponential function, Lin Biao expands and enriches the matrix exponential functions eA and eAt, and further gives a general calculation method to prove the matrix exponential function by using the matrix similarity principle (Lin, 2010). In order to clarify the nature of the matrix index, Zheng Xingzhong further studied the Kronecker product (and), the Khatri-Rao product (and), and the acy-Singh product (and), and discussed the matrix exponential function on this basis. The particularity of transposing (Zheng and Ren, 2011). Liu Yuyun et al. In the study of the calculation of matrix exponential function, it is mentioned that the matrix exponential functions exp(A) and exp(At) have been widely used in differential equations and modern system control. Among them, the integral representation of each derivative in the matrix exponential function is the basis of the application of the exponential functions exp(A) and exp(At) in actual production. The exponential function exp(A) is obtained mainly by the method and skill of the matrix column expansion operation. The first-order directional derivative of exp(At) (Liu et al., 2009). Yu Liya pointed out that there are many methods for calculating the matrix exponential function, including differential equation method, polynomial method and matrix decomposition method. Both methods have certain limitations, both theoretically and computationally. Both methods have certain limitations, both theoretically and computationally. Therefore, Yu Liya has developed a new method for calculating the matrix exponential function from the linear homogeneous homogeneous differential equation and the Hamilton-Kelley theorem. This method makes the calculation of the matrix function more practical (Yu, 2010).

1.2 Research Purposes

At present, with the extensive use of mechanical design and oceanography in real life, the method of index integral method of computational methods has been widely recognized by scholars, and has obtained rich research results. After continuous research and daily application, scholars have an important role in the mechanical design and oceanographic calculations, and further pointed out that the exponential integration method is an effective numerical method for solving semilinear differential equations (Chen, 2011). Specifically, the exponential property-like matrix function directly solves the efficiency of the exponential integration method by efficiently solving with the vector product. At the same time, the specific solution process of ordinary differential equations has been the research field of computational direction in data, and many mature algorithms have been

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formed. Among them, these algorithms have achieved good results in solving non-rigid differential equations. The main performance is that implicit integration can effectively overcome the inefficiency of the rigid numerical solution method. However, it is difficult to solve the problem, and it needs to be solved by means of an exponential class matrix function to reduce the amount of calculation. In this context, it is important to study the validity of the matrix exponential function and the vector product.

2. Theoretical Overview

At present, the study of exponential matrix functions has been widely used in social production and has attracted the attention of most scholars. Among them, the algorithm for solving the matrix index and vector product has achieved great results in many scholars and a wide range of practical applications (Cai et al., 2014). At the same time, for the exponential function with large sparse matrix, the Krylov subspace method is generally used to solve the vector product of the matrix exponential function. In the specific solution process, the vector product solving method of matrix exponential function has been greatly improved, and the standard subspace method has been improved and optimized in continuous research and application (Guo, 2011). exponential function is optimized to a large extent, the application and research of the subspace method of the matrix exponential function $e^A b$ is still in its infancy, and there is no systematic use and specific description.

Aiming at this situation, this paper constructs a block-based Krylov subspace method to calculate the matrix exponential function. Compared with the Krylov subspace algorithm of single vector, the block self-space method constructed in this paper shows a big advantage when developing a matrix with heavy eigenvalues and close eigenvalues, which can ensure that the eigenvalues are higher. At the same time, when calculating the block subspace, it can solve the original reduction original problem that cannot be solved by the unidirectional quantum space, and can avoid the unnecessary operation steps of the matrix and vector product to a certain extent. At the same time, the block subspace can reduce the instability factor in the operation process and ensure the accuracy of the calculation results. In the specific calculation process, the block Krylov subspace mainly performs a low-dimensional processing on a multi-dimensional matrix A, and then approaches the initial problem by a low-order matrix index. In the calculation process, in order to make the calculation process more reasonable, the ODEs system is used in the calculation process to construct the block Krylov subspace method.

3. Effective Calculation of Matrix Exponential Function and Vector Product

3.1 Block Krylov Subspace Method

To represent a larger order of matrix exponents and vector product forms, the following theorem can be used to provide research motivation for the development of Krylov subspace methods, as shown below.

Hypothesis $A \in C^{n\times n}, \quad W = [\omega_1, \omega_2, \cdots, \omega_p] \in C^{n\times p}, \quad \tau \in C$,and $\tilde{A} = \begin{pmatrix} A & W \\ O & J \end{pmatrix} \in C^{(n+p)\times(n+p)}, \quad J = \begin{pmatrix} O & I_{p-1} \\ 0 & O \end{pmatrix} \in R^{p\times p}$

When $X = \varphi_t(t, \tilde{A}), \quad l \geq 0$, then there is

$$X = (1:n, n+j) = \sum_{k=1}^{j} t^k \varphi_t + k(t\tilde{A})\omega_{j-k+1}, \quad j = 1:p \qquad (1)$$

In formula (1), each variable element takes the following condition $l = 0, \quad \omega_{p-k+1} = b_k$, and

$$X=(1:n,n+p)=[I_n,O]\varphi_0(t\tilde{A})e_{n+p}=\sum_{k=1}^{p}t^k\varphi_k(tA)b_k \qquad (2)$$

$$\text{Further get}\sum_{k=0}^{p}t^k\varphi_k(tA)b_k=[I_n,O]e^{t\tilde{A}}\begin{pmatrix}b_0\\ e_p\end{pmatrix} \qquad (3)$$

Let $t=1$, then the left side of equation (3) is to solve the problem of equation (1). At this time, the solution of the problem (1) can be transformed into a matrix exponential with a larger order and a vector product. In this way, the algorithm for calculating the larger-order matrix exponent and the vector product can calculate the problem (1). At the same time, some scholars will use the relationship between formula (1) and formula (2), using the stage Taylor of *exp(A)* to show the right end term of approximation formula (3), thus avoiding the operation between a large number of matrices, and the matrix inverse operation, This implements a matrix and vector product operation.

3.2 Forward Error Analysis and Posterior Error Estimation

Algorithm 1 is first subjected to forward error analysis, followed by reliable a posteriori error estimation as a criterion for algorithm termination.

Solve the block Krylov subspace algorithm:

Enter: $A,B=\left[b_0,b_1,...b_p\right] andt.$

$b_1=b_1+Ab_0$

$B(:,2:p+1)$ 'S QR Decomposition: $B(:,2:p+1)=V_1R$

$for \quad m=1,2,...do$

Calculate the $Arnoldi/Lanczos$ Decomposition of $k_m(A,V_1)$

Form: $\overline{H_m}:W(:,i)=(E_1R)(:,p-i+1),i=1,2,...p$

Calculation $u=\left[I_{mp},O\right]e^{t\overline{Hm}}e_{p(m+1)}$

Calculation $y_m=v_mu+b_o$

$end \;\; for$

First, forward error analysis. Based on the previous theoretical analysis, the analytical solution of the system is assumed to be the formula (4). Among them, y is an exact solution.

$$y(t)=[V_m,O]e^{t_m}e_p(m+1) \qquad (4)$$

At this point, it is assumed that the matrix $\tilde{H}_m$ index is completed by any suitable small- and medium-scale matrix index algorithm. When the $u(t)$ value is obtained, it is brought into the formula (4) to obtain the following formula (5), where y_m is a numerical solution.

$$y_m(t)=V_mu(t)=[V_m,O]e^{tm}ep(m+1) \qquad (5)$$

Subsequently, the formulas (4) and (5) are subtracted correspondingly to obtain the precision error of $y_m(t)$.

$$e_m(t)=y(t)-y_m(t) \qquad (6)$$

Second, the posterior error estimate. In order to determine whether the approximate solution A satisfies the accuracy requirement, it is necessary to establish a reliable backward error estimate as the algorithm termination criterion.In order to further solve the initial value problem, the expressions of the systematic error $r_m(t)$ and the error $\varepsilon_m(t)$ generated by the approximate solution y_m are extracted. At the same time, numerical experiments and theoretical analysis show that the error estimates extracted from these expressions can truly reflect the law of error variation. The specific formula is as shown in (5).

$$e_m^1 = t \| V_{m+1} H_{m+1}, E_m^T \varphi_1 e_p (m+1) \| \quad (5)$$

In order to simplify the calculation process, a simple matrix error statistic formula (6) can be obtained by replacing the function φ_1 with a matrix exponent.

$$e_m^2 = t \| V_{m+1} H_{m+1}, m E_m^T \varphi_1 e_p (m+1) \| \quad (6)$$

3.3 Numerical Experiment

In order to verify the validity of the algorithm and the backward error estimate, it is necessary to carry out numerical experiments. The test environment is Windows 7 system, the processor is i7-8400, 3.0GHz, 8G memory. The MATLAB software is used to measure the accuracy of the algorithm by the relative error formula (7). If not specified, the experimental exact solution is calculated by MATLAB's own function ode45.

$$Error = \frac{\| y - y_m \|}{\| y \|} \quad (7)$$

For convenience of presentation, we estimate the relative a posteriori error using $\varepsilon_m^1, \varepsilon_m^2, \varepsilon_m^3$, and ε_m^4, , respectively, and derive the following formula (8).

$$\varepsilon_m^i := \frac{\varepsilon_m^i}{\| y \|}, \ i = 1, 2, 3, 4. \quad (8)$$

The specific inspection process is as follows. We use Algorithm 1 to calculate the value of $\varphi_0(tA)b_0 + \varphi_1(tA)b_1 + ... + \varphi_5(tA)b_5$ at $t = \pm 1$.

When $b_0 = 0, b_i = e_i, i = 1, 2, ..., 5$, select 2 common test matrices. The first is the Markovchains matrix, which is an asymmetric matrix of 11264 non-zero elements. The second matrix is a block-symmetric tridiagonal matrix of order 9801, which is obtained by finite difference method and can be generated by MATLAB commands.

Through calculation, Algorithm 1 has a super linear convergence speed, which can achieve higher calculation accuracy. At the same time, the backward error estimation is more effective and consistent with the real trend. In some environments, these error estimates overlap with real errors. Therefore, the accuracy of the modified algorithm 1 is high and has certain practicality.

4. Conclusion

In summary, this paper introduces the related theory of exponential class matrix function, further constructs the block Krylov subspace, and studies the effective calculation between matrix exponential function and vector product. In the specific research process, the data was tested mainly by forward error analysis, posterior error estimation, and format correction. Finally, by optimizing the dimension of the Krylov subspace, the effective calculation algorithm can meet the precision requirements of the matrix exponential function with a minimum workload.

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